

Valuation of Currency Options using Higher Order Moments

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Abstract

The currency exchange rates under consideration were not lognormally distributed for February 1989 to February 2004, violating one of the assumptions of the Black-Scholes model. We incorporate higher order moments via an Edgeworth series expansion applied directly to the probability density function. Option values and comparative statics (the “Greeks”) are computed and compared with those obtained using the Black-Scholes method. The results indicate significant differences, which has major implications for hedging and risk management. The pricing difference is largest in magnitude for out-of-the money call options and at-the-money put options and varies linearly with kurtosis and nonlinearly with skewness. The “Greeks” also display considerable differences under the two pricing methodologies. When skewness and kurtosis vary simultaneously, the former dominates for both in-the-money and out-of-the money puts and calls.

Key words: Exchange rates, Currency options, Black and Scholes, Non-normal, Skewness, Kurtosis.

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I. Introduction

A wide collection of financial models and theories are based on the assumption that security returns are normally distributed. Aside being widely understood, the normal distribution has the desirable property that it can be fully described by its first two moments: the mean and the variance. Skewness (the third moment) and kurtosis (the fourth moment) measure the degree of asymmetry and the degree of peakedness, respectively. For a normal distribution, skewness is equal to zero and kurtosis is equal to three.

Although the normality assumption is widely used in the financial literature, it is now a well-known empirical finding that the distribution of exchange rates exhibits a kurtosis different from 3 (the so called “fat tails”) and non-zero skewness, implying that the probability of observing extreme events is greater than that under the normality assumption. In a study covering eight major currencies from 1974 to 1984, Aggarwal (1990) finds significant departures from the normality assumption¹. Sterge (1989) uncovers the same pattern in the frequency of extreme observations in the returns of fixed income securities².

Since 1973, the Black-Scholes-Merton framework³ has been the primary tool in option pricing. Although a very important breakthrough in pricing derivatives, pivotal for the tremendous growth of financial engineering in the 1980’s and the 1990’s, certain assumptions underlying the Black-Scholes pricing model are not supported empirically. If the underlying asset prices over the option’s life are lognormally distributed, underlying returns will have a normal distribution with constant variance. Several research studies document that the Black and Scholes paradigm ignores asymmetries, as well as larger probabilities of extreme returns. Most notably, Macbeth and Merville (1979) find that in-the-money options are undervalued, whereas out-of-the-money options are overvalued in the Black-Scholes framework.

¹ Early studies include Mandelbrot (1963) and Fama (1965).

² Sterge examined short and long-term fluctuations in Treasury bonds, 10-year Treasury notes, and Eurodollar futures prices.

³ “The pricing of options and corporate liabilities” by F. Black and M. Scholes, 1973; and “Theory of rational option pricing” by R.C. Merton, 1973.

Merton (1976) makes one of the first attempts to correct the shortcomings of the Black-Scholes framework. He incorporates a Poisson-driven jump process allowing for discontinuities in the stock price paths. However, this model requires estimation of the number of jumps per unit of time, and the percentage of total volatility explained by the jumps, so it is quite difficult to parameterize. Jarrow and Rudd (1982) try to account for non-normal stock price returns by adjusting the Black-Scholes price with a term depending on the second and higher order moments of the underlying diffusion process. The series of adjustment terms in the Jarrow and Rudd approach stems directly from the difference between the true and the approximated distribution of the second, third and fourth moments. While the main contribution of this approach is to provide a methodology for pricing options based on arbitrary stock price diffusion processes, its implementation and understanding are quite computationally challenging.

More recently, Corrado and Su (1996) employ a Gram-Charlier series expansion to the normal density function, and incorporate skewness and kurtosis adjustment terms in the Black-Scholes framework. This model minimizes the sum of squared errors to solve for the implied volatility, skewness and kurtosis measures, and the findings are consistent with the empirical reality of non-normality.

Rubinstein (1998) applies an Edgeworth series expansion in a binomial framework, considerably reducing the computational complexities introduced by higher order moments. Tian (1998) incorporates both skewness and kurtosis, accommodating both the “fat tails” and the “peakedness” generally observed in security returns, and indicates mispricing of options occurs whenever the non-normality of returns is ignored. However, both models are limited in that they accommodate only a limited number of skewness-kurtosis combinations.

The focus of this paper is to test the option pricing model of Young, Jabbour, Kramin, and Kramin (2002) who incorporate higher-order moments via Edgeworth series expansion applied directly to the

probability density function. Using a data sample of S&P index returns from December 1989 to November 2001, they find that the option prices derived from the Black-Scholes formula are below the prices derived from the Edgeworth expansion for low strikes and above Edgeworth prices for high strikes, both for puts and for calls. As opposed to Young, Jabbour, Kramin, and Kramin (2002), we use foreign exchange data (Yen/USD exchange rate) which is known to have different characteristics than index returns. Also, we enhance their methodology by performing a bivariate sensitivity analysis of the pricing differential between the Black-Scholes model and the Edgeworth expansion. The rest of the paper is structured as follows: section II presents the model; section III discusses the results; section IV performs a sensitivity analysis to demonstrate how different degrees of skewness and kurtosis affect the difference in Black-Scholes pricing as opposed to the pricing obtained via Edgeworth expansion; and section V concludes the study.

II. The model

In a risk neutral framework, the price of any European call option today is the expected value of the instrument at maturity, discounted at the risk free rate

$$C_0 = e^{-rT} E^*[C_T] \quad (1)$$

where E^* denotes a risk neutral framework, r is the risk free rate, T represents the maturity of the option, K is the exercise price, and C_T is the payoff of the option at maturity ($C_T = \max \{S_T - K, 0\}$.)

In order to value currency options, we define the spot exchange rate S_t as the value of one unit of foreign currency measured in the domestic currency. We assume that S_t follows:

$$dS_t = (r - r_f) S_t dt + \sigma S_t dW_t \quad (2)$$

where r is the domestic risk-free rate, r_f is the foreign risk-free rate, σ is the instantaneous volatility of the exchange rate, S_0 is the current

spot rate, and dW_t is a Wiener process. In the Black-Scholes framework, the option values are given by:

$$\text{Call:} \quad C = \frac{e^{-rT}}{\sqrt{2\pi}} \int_0^a (e^{b+m} - K) e^{-\frac{1}{2}t^2} dt \quad (3)$$

$$\text{Put:} \quad P = \frac{e^{-rT}}{\sqrt{2\pi}} \int_a^\infty (K - e^{b+m}) e^{-\frac{1}{2}t^2} dt \quad (4)$$

$$\text{where } a = \frac{\ln(K) - m}{s}, \quad s = \sigma \sqrt{T}, \quad \text{and } m = \ln(S_0) + (r - r_f) T - \frac{\sigma^2}{2} T.$$

In order to incorporate the effect of higher-order moments, we employ an Edgeworth series expansion as in Young, Jabbour, Kramin and Kramin (2002). Given a predetermined level of skewness ξ and kurtosis κ , we can write the probability density function as

$$f(t) = 1 + \frac{1}{6} \xi (t^3 - 3t) + \frac{1}{24} (\kappa - 3)(t^4 - 6t^2 + 3) + \frac{1}{72} \xi^2 (t^5 - 10t^3 + 15t) \quad (5)$$

If $\xi = 0$ and $\kappa = 3$, then $f(t) = 1$. In such case, option prices are consistent with the Black-Scholes paradigm and the probability density function is that of a standard normal random variable. Otherwise, we can rewrite equations (3) and (4) as

$$C_E = \frac{e^{-rT}}{\sqrt{2\pi}} \int_0^a (e^{b+m} - K) e^{-\frac{1}{2}t^2} f(t) dt \quad (6)$$

$$P_E = \frac{e^{-rT}}{\sqrt{2\pi}} \int_a^\infty (K - e^{b+m}) e^{-\frac{1}{2}t^2} f(t) dt \quad (7)$$

where the subscript E denotes the fact that we are dealing with values obtained using the Edgeworth expansion. These integrals are evaluated numerically as well as the “Greeks”. Option prices and the “Greeks” provide insight on biases introduced by non-normal underlying returns.

III. Results

We use the exchange rate Yen/USD for a period from 01/01/1984 to 01/01/2004. Daily returns, the sample mean, standard deviation, skewness and excess kurtosis are calculated as follows:

$$r_i = \ln \frac{S_t}{S_{t-1}} \quad (8)$$

$$\bar{r} = \frac{1}{n} \sum_{i=1}^n r_i \quad (9)$$

$$s = \sqrt{\frac{n \sum_{i=1}^n r_i^2 - \left(\sum_{i=1}^n r_i \right)^2}{n(n-1)}} \quad (10)$$

$$\xi = \frac{n}{(n-1)(n-2)} \sum_{i=1}^n \left(\frac{r_i - \bar{r}}{s} \right)^3 \quad (11)$$

$$K_{\text{excess}} = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^n \left(\frac{r_i - \bar{r}}{s} \right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)} \quad (12)$$

where r_i is the return for day i , \bar{r} is the mean of the return series, n is the number of observations, s is the sample standard deviation, ξ is the skewness, and K_{excess} is the excess kurtosis.

Table 1 provides descriptive statistics for the sample of foreign exchange returns for the Yen/USD exchange rate return for the period of 01/01/1984 to 01/1/2004. Specifically, the table reports the mean, minimum, and maximum values of skewness, excess kurtosis and standard deviation⁴, computed on a 1-month, 3-month, 6-month and 12-month rolling basis. The mean skewness decreases monotonically from negative 0.178, when computed on a 1-month rolling period, to a negative 0.5 when computed on a 12-month rolling basis. The mean annualized standard deviation averages about 11 percent regardless of the rolling period, but features considerable dispersion, from a minimum of 0.032 to a maximum of 0.351, both of which correspond to the 1-month rolling period.

The series is also highly leptokurtic: the excess kurtosis is 3.9 when computed on a 12-month rolling basis. Our findings are consistent with Rubinstein (1998) and Young, Jabbour, Karmin, and Karmin (2002) who found that S&P 500 index options market is left-skewed and leptokurtic. However, the evidence we report shows that exchange rate returns are more leptokurtic and left-skewed than index returns, which suggests the non-normality becomes more of a problem when dealing with foreign exchange options rather than index options. For example, Young, Jabbour, Karmin, and Karmin (2002) report values ranging from -0.0151 to -0.1847 for skewness and 0.5608 to 1.6842 for excess kurtosis, respectively, which are considerably lower than our results for the Yen/USD exchange rate.

⁴ Daily standard deviations are annualized.

Table 1. Yen/USD exchange rate returns skewness and excess kurtosis (01/01/1984 to 01/1/2004).

	Mean	Maximum	Minimum
1-month rolling skewness, kurtosis and standard deviation (24 observations per period)			
Skewness	-0.178	2.554	-3.565
Excess kurtosis	1.319	14.905	-1.450
Standard Deviation	0.106	0.351	0.032
Number of observations: 3889			
3-month rolling skewness, kurtosis and standard deviation (64 observations per period)			
Skewness	-0.340	1.899	-3.749
Excess kurtosis	2.246	21.881	-0.742
Standard Deviation	0.109	0.259	0.061
Number of observations: 3849			
6-month rolling skewness, kurtosis and standard deviation (128 observations per period)			
Skewness	-0.427	1.592	-3.724
Excess kurtosis	2.969	27.663	0.435
Standard Deviation	0.110	0.220	0.071
Number of observations: 3786			
12-month rolling skewness, kurtosis and standard deviation (256 observations per period)			
Skewness	-0.499	0.596	-2.100
Kurtosis	3.852	16.371	0.123
Standard Deviation	0.111	0.187	0.071
Number of observations: 3657			

Tables 2 and 3 compare the values, as well as the “Greeks” for call and put option values obtained using both Black-Scholes and the Edgeworth frameworks. The assumptions include an underlying value

of Yen/USD foreign exchange rate of 105 Yen/USD, strikes ranging from 100 to 110, a domestic interest rate⁵ for Japan's 3-month government bonds of 0.0675 percent; a foreign interest rate for US 3-month Treasury bills rate of 0.93 percent, and a volatility of 10.9 percent (as computed in Table 1 on a 3-month rolling basis). In order to price the options under the Edgeworth expansion framework we used an excess kurtosis of 2.246 and a skewness of negative 0.34, also corresponding to a 3-month rolling basis. The values for the underlying exchange rate and both the domestic and the foreign risk-free rates are as of February 2004.

Table 2. Comparison of call option prices and the Greeks

Strike	Call Value		Difference	%	Delta		Difference	%
	BS	Edge	Edge-BS	Difference	BS	Edge	Edge-BS	Difference
100	5.380	5.400	0.020	0.37%	0.809	0.862	0.053	6.53%
101	4.611	4.572	-0.039	-0.85%	0.756	0.815	0.059	7.75%
102	3.901	3.800	-0.101	-2.59%	0.696	0.754	0.057	8.22%
103	3.255	3.097	-0.157	-4.84%	0.632	0.680	0.049	7.70%
104	2.677	2.475	-0.202	-7.54%	0.563	0.597	0.034	5.98%
105	2.169	1.940	-0.228	-10.53%	0.494	0.509	0.015	2.97%
106	1.730	1.495	-0.235	-13.59%	0.425	0.420	-0.006	-1.30%
107	1.358	1.136	-0.222	-16.37%	0.359	0.335	-0.024	-6.68%
108	1.049	0.855	-0.193	-18.42%	0.298	0.260	-0.038	-12.87%
109	0.796	0.644	-0.153	-19.17%	0.242	0.195	-0.047	-19.42%
110	0.595	0.488	-0.107	-17.96%	0.193	0.143	-0.050	-25.75%

⁵ We take the prospective of an investor based in Japan.

Strike	Vega		Difference	%	Theta		Difference	%
	BS	Edge	Edge-BS	Difference	BS	Edge	Edge-BS	Difference
100	14.151	11.763	-2.388	-16.88%	2.348	1.780	-0.569	-24.21%
101	16.351	13.716	-2.635	-16.12%	2.877	2.249	-0.628	-21.82%
102	18.257	15.657	-2.600	-14.24%	3.347	2.728	-0.619	-18.50%
103	19.719	17.309	-2.410	-12.22%	3.724	3.155	-0.570	-15.29%
104	20.619	18.404	-2.215	-10.74%	3.983	3.469	-0.513	-12.89%
105	20.894	18.753	-2.141	-10.25%	4.106	3.626	-0.480	-11.69%
106	20.537	18.289	-2.248	-10.95%	4.091	3.606	-0.485	-11.85%
107	19.595	17.084	-2.511	-12.81%	3.946	3.420	-0.525	-13.31%
108	18.165	15.323	-2.842	-15.65%	3.690	3.105	-0.585	-15.84%
109	16.374	13.255	-3.119	-19.05%	3.350	2.713	-0.637	-19.02%
110	14.363	11.138	-3.225	-22.45%	2.956	2.298	-0.658	-22.25%

Strike	Gamma		Difference	%	Rho		Difference	%
	BS	Edge	Edge-BS	Difference	BS	Edge	Edge-BS	Difference
100	0.047	0.039	-0.008	-17.97%	19.904	21.291	1.387	6.97%
101	0.054	0.052	-0.002	-4.28%	18.702	20.258	1.556	8.32%
102	0.061	0.066	0.005	8.07%	17.313	18.850	1.537	8.88%
103	0.066	0.077	0.012	18.12%	15.771	17.095	1.324	8.40%
104	0.069	0.086	0.017	25.25%	14.124	15.063	0.939	6.65%
105	0.069	0.090	0.020	29.05%	12.424	12.867	0.443	3.57%
106	0.068	0.088	0.020	29.35%	10.728	10.639	-0.089	-0.83%
107	0.065	0.082	0.017	26.17%	9.088	8.508	-0.580	-6.38%
108	0.060	0.072	0.012	19.84%	7.551	6.586	-0.965	-12.78%
109	0.054	0.060	0.006	10.82%	6.151	4.947	-1.204	-19.57%
110	0.048	0.048	0.000	0.42%	4.913	3.627	-1.286	-26.17%

Table 3. Comparison of put option prices and the Greeks.

Strike	Put Value		Difference	%	Delta		Difference	%
	BS	Edge	Edge-BS	Difference	BS	Edge	Edge-BS	Difference
100	0.607	0.628	0.021	3.44%	-0.189	-0.136	0.053	-28.04%
101	0.838	0.800	-0.038	-4.57%	-0.242	-0.183	0.059	-24.26%
102	1.127	1.028	-0.100	-8.86%	-0.301	-0.244	0.057	-19.02%
103	1.481	1.325	-0.156	-10.56%	-0.366	-0.317	0.049	-13.29%
104	1.903	1.702	-0.201	-10.55%	-0.434	-0.401	0.034	-7.76%
105	2.395	2.167	-0.227	-9.50%	-0.504	-0.489	0.015	-2.92%
106	2.956	2.722	-0.234	-7.92%	-0.572	-0.578	-0.005	-0.96%
107	3.584	3.362	-0.222	-6.18%	-0.638	-0.662	-0.024	-3.75%
108	4.278	4.082	-0.196	-4.59%	-0.700	-0.738	-0.038	-5.47%
109	5.022	4.870	-0.152	-3.03%	-0.755	-0.802	-0.047	-6.22%
110	5.820	5.714	-0.106	-1.82%	-0.805	-0.854	-0.050	-6.18%

Strike	Vega		Difference	%	Theta		Difference	%
	BS	Edge	Edge-BS	Difference	BS	Edge	Edge-BS	Difference
100	14.151	11.790	-2.361	-16.68%	3.255	2.692	-0.563	-17.28%
101	16.351	13.741	-2.610	-15.96%	3.783	3.160	-0.622	-16.45%
102	18.257	15.682	-2.575	-14.10%	4.252	3.639	-0.614	-14.43%
103	19.719	17.334	-2.385	-12.09%	4.629	4.065	-0.564	-12.18%
104	20.619	18.429	-2.190	-10.62%	4.887	4.379	-0.508	-10.40%
105	20.894	18.778	-2.116	-10.13%	5.010	4.535	-0.475	-9.47%
106	20.537	18.314	-2.223	-10.82%	4.994	4.514	-0.479	-9.60%
107	19.595	17.109	-2.486	-12.69%	4.848	4.328	-0.520	-10.72%
108	18.165	15.348	-2.817	-15.51%	4.591	4.012	-0.579	-12.61%
109	16.374	13.280	-3.094	-18.90%	4.251	3.619	-0.632	-14.86%
110	14.363	11.163	-3.200	-22.28%	3.856	3.204	-0.652	-16.92%

Strike	Gamma		Difference	%	Rho		Difference	%
	BS	Edge	Edge-BS	Difference	BS	Edge	Edge-BS	Difference
100	0.047	0.039	-0.008	-17.97%	-5.092	-3.704	1.388	27.27%
101	0.054	0.052	-0.002	-4.27%	-6.544	-4.987	1.556	23.78%
102	0.061	0.066	0.005	-8.07%	-8.183	-6.645	1.538	18.80%
103	0.066	0.077	0.012	-18.13%	-9.974	-8.650	1.324	13.28%
104	0.069	0.086	0.017	-25.25%	-11.872	-10.932	0.940	7.92%
105	0.069	0.090	0.020	-29.05%	-13.822	-13.377	0.445	3.22%
106	0.068	0.088	0.020	-29.35%	-15.768	-15.856	-0.088	-0.56%
107	0.065	0.082	0.017	-26.22%	-17.658	-18.236	-0.578	-3.27%
108	0.060	0.072	0.012	-19.90%	-19.445	-20.409	-0.964	-4.96%
109	0.054	0.060	0.006	-0.82%	-21.094	-22.297	-1.203	-5.70%
110	0.048	0.048	0.000	-0.42%	-22.582	-23.867	-1.285	-5.69%

In general, as shown by the evidence in Tables 2 and 3, option prices computed using the Black-Scholes methodology are different than those obtained using the Edgeworth expansion. The percentage difference between the Edgeworth call option price and the Black-Scholes prices decreases almost monotonically from in-the-money calls (0.37 percent) to out-of-the-money calls (-17.96 percent). For all but one simulation run (strike price of 100 Yen/USD), the Edgeworth call price is lower than the Black and Scholes call price. At-the-money (strike price of 105Yen/USD), the Edgeworth price is lower roughly by 11 percent, but the difference increases fast as we move out-of-the money (approximately -18 percent for a strike of 108 Yen/USD).

With regard to puts, Edgeworth prices are again below the Black and Scholes prices. However, the percentage difference is not monotonic. Out-of-the money (strike price of 100 Yen/USD), the Edgeworth price is by 3.5 percent higher than the Black-Scholes price. For a strike of 101 Yen/USD, the difference between the Edgeworth put price and the Black-Scholes price becomes negative (-4.57 percent) and decreas-

es to approximately -10 percent for at-the-money options. As the option moves in-the-money, the difference decreases monotonically to -1.82 percent for a strike of 110 Yen/USD. Thus, as opposed to call options where we experience the highest magnitude in price differences for out-the-money options, put options display the largest pricing difference when they are slightly in the money.

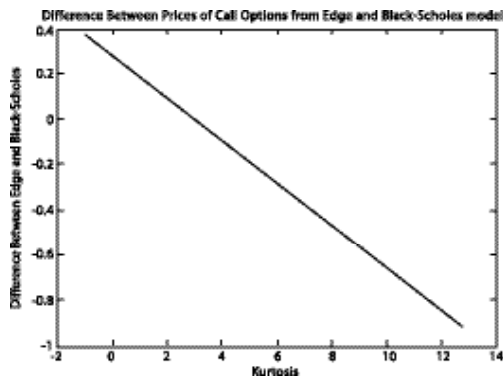
The “Greeks” also differ significantly in magnitude under the two methodologies. For example, the difference between delta for Edgeworth calls and delta for Black-Scholes calls is 6.53 percent for in-the-money calls (100 Yen/USD), 2.97 percent for at-the-money calls (105 Yen/USD) and -25.75 percent for out-of-the-money calls (110 Yen/USD). The difference between delta for Edgeworth puts and delta for Black-Scholes puts is -28.04 percent for out-of-the-money puts (100 Yen/USD), -2.92 percent for at-the-money puts (105 Yen/USD), and 6.18 percent for in-the-money puts (110 Yen/USD). In terms of magnitude, the difference seems to be most significant for out-of-the-money puts and calls (more than 25 percent), and less significant for at-the-money calls and puts (about 3 and respectively -3 percent). The magnitude difference is considerable for the rest of the “Greeks” too, going as high as 29 percent for Gammas computed for both at-the-money calls and puts.

IV. Sensitivity analysis

IV.a. Univariate sensitivity analysis

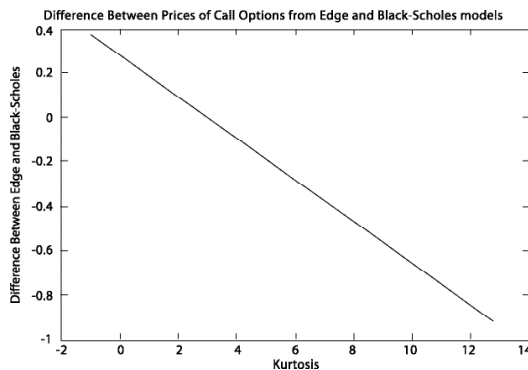
A univariate sensitivity analysis was performed for the difference between Edgeworth and Black-Scholes prices, for at-the-money, in-the-money and out-of-the-money call and put options. First, skewness was fixed at 0 and excess kurtosis varied between 1 and 13 (we use 70 intermediate values). Figures 1, 2, and 3 depict this difference for at-the-money, in-the-money, and out-of-the money call options, with kurtosis varying, while skewness is kept fixed at 0.

Figure 1: at-the-money call options⁶ ($X = 105, S = 105$)



Slope = -10

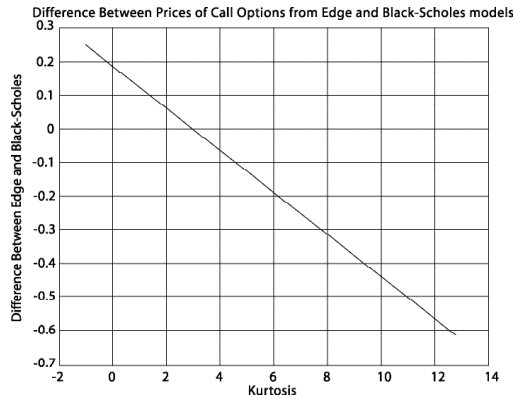
Figure 2: in-the-money call options ($X = 102.5, S = 105$)



Slope = -10

⁶ The horizontal axis represents kurtosis, not excess kurtosis.

Figure 3: out-of-the-money call options (X = 107.5, S = 105)



Slope = -15

Figures 4, 5, 6 depict the pricing differential for the same at-the-money, in-the-money, and out-of-the-money call options, with varying degrees of skewness and fixed kurtosis.

Figure 4: at-the-money call options (X = 105, S = 105)

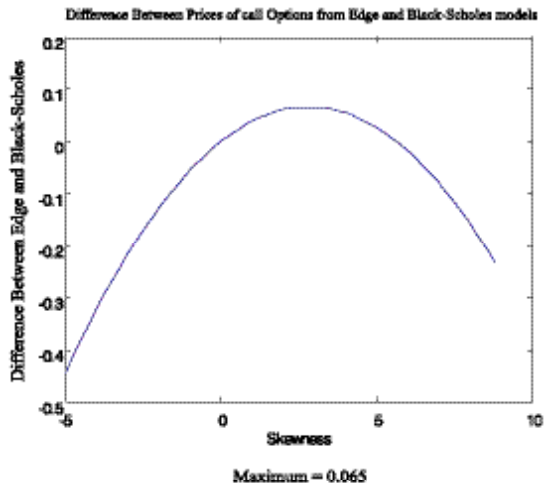


Figure 5: in-the-money call options ($X = 102.5, S = 105$)

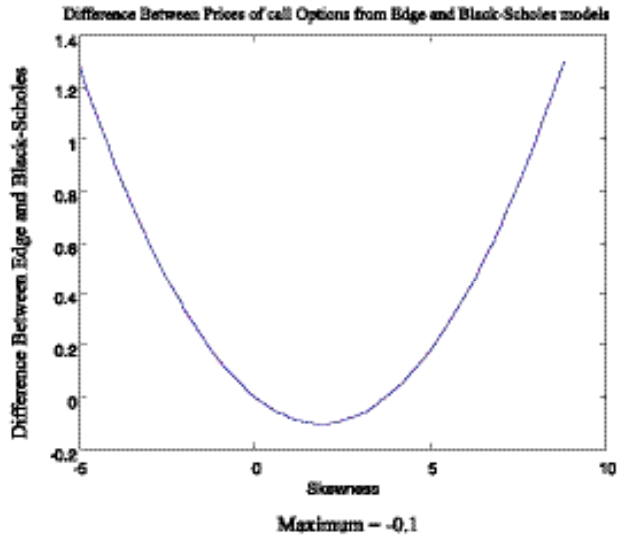
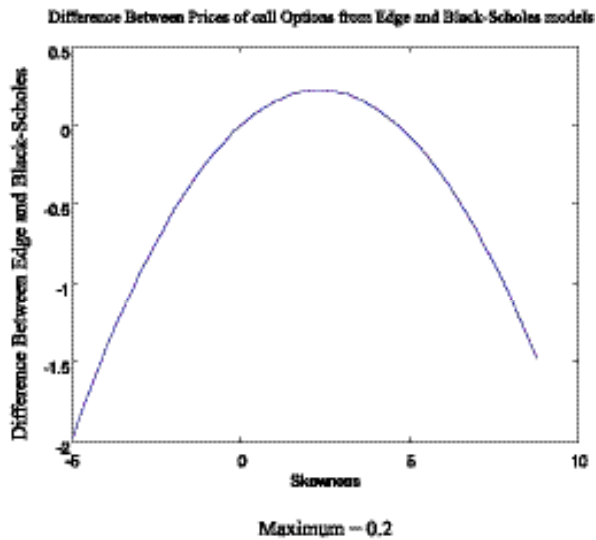
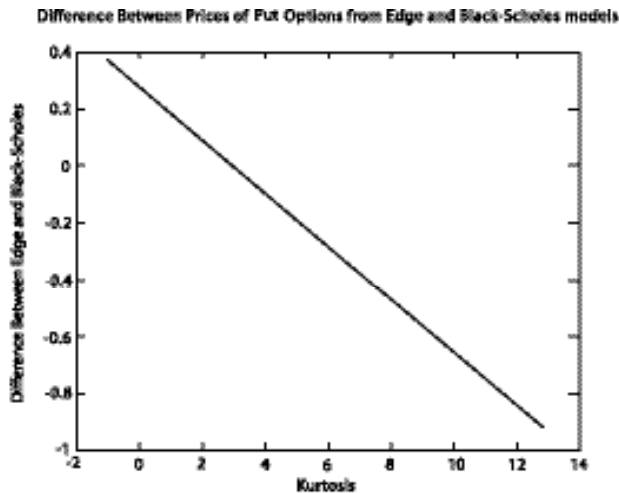


Figure 6: out-of-the-money call options ($X = 107.5, S = 105$)



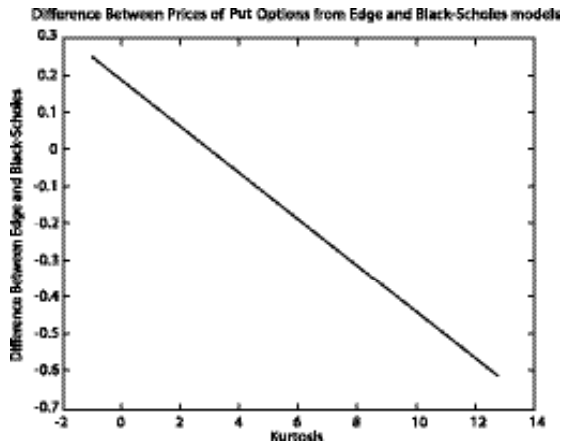
The univariate sensitivity analysis revealed that the difference between option prices obtained from the Edgeworth and Black-Scholes models is linearly related to kurtosis for both call options and put options, out-of-the-money, at-the-money and in-the money: as kurtosis is increased, the difference between Edgeworth prices and Black-Scholes prices becomes more negative. The difference in pricing is nonlinearly related to skewness. For at-the-money and out-of-the-money call options, as well as for at-the-money and in-the-money put options, the difference increases monotonically to a maximum, then decreases monotonically. For in-the-money calls and out-of-the-money puts, the difference is U-shaped: it decreases monotonically to a minimum then increases again. Most importantly, the difference grows in magnitude as the skewness becomes more and more negative.

Figure 7: at-the-money put options (X = 105, S = 105)



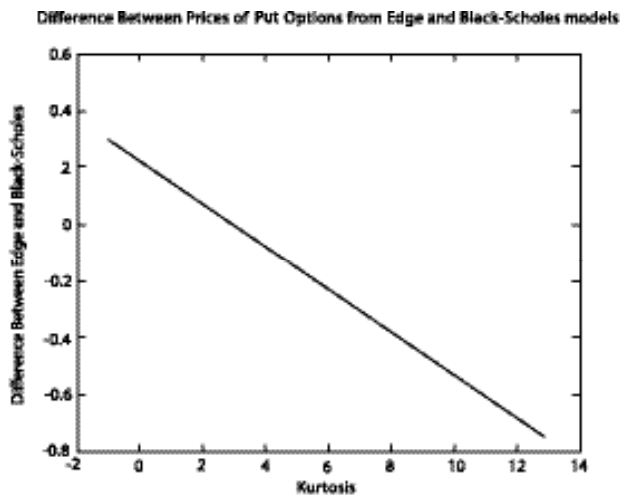
Slope = -10

Figure 8: in-the-money put options ($X=107.5, S = 105$)



Slope = -15

Figure 9: out-of-the money put options ($X=102.5, S = 105$)



Slope = -10

Figure 10: at-the-money put options ($X = 105, S = 105$)

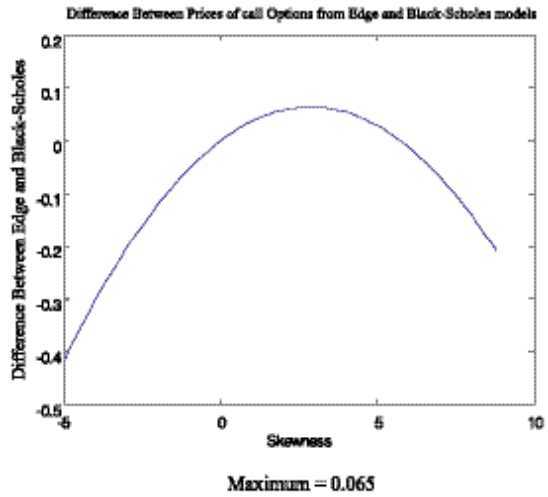
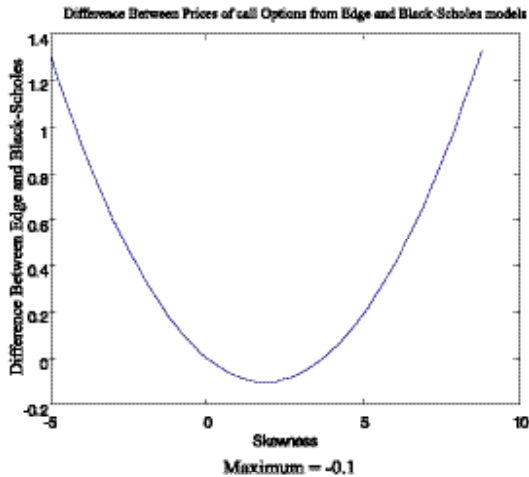


Figure 11: in-the-money put options ($X=107.5, S = 105$)



Figure 12: out-of-the money put options ($X=102.5, S = 105$)



IV.b. Bivariate sensitivity analysis

The bivariate sensitivity analysis produced some interesting results: it seems that when skewness and kurtosis vary together, they both have an effect on the pricing difference only for at-the-money calls and at-the-money puts. In the remaining cases, the skewness effect clearly dominates the kurtosis effect.

Figure 13: at-the-money call options ($X = 105, S = 105$)

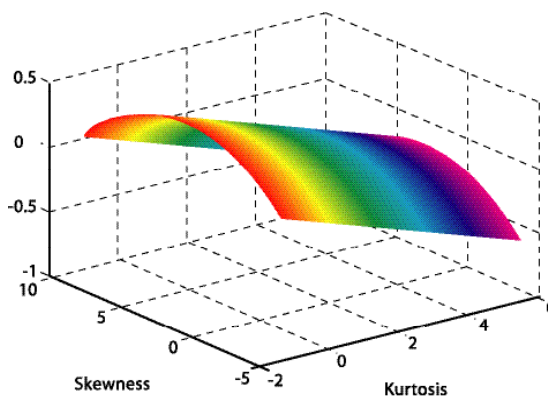


Figure 14: in-the-money call options ($X = 102.5, S = 105$)

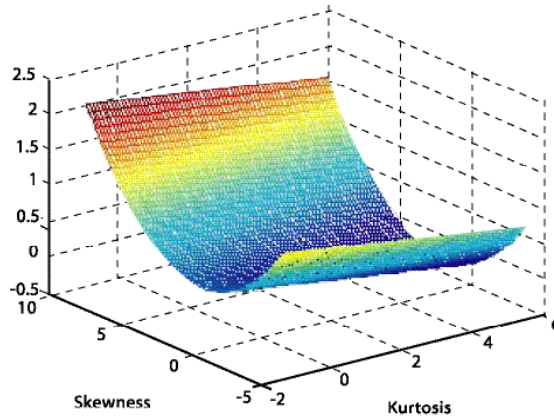


Figure 15: out-of-the-money call options ($X = 107.5, S = 105$)

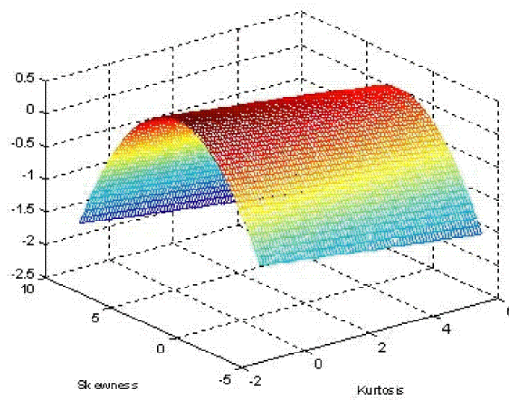


Figure 16: at-the-money put options ($X = 105, S = 105$)

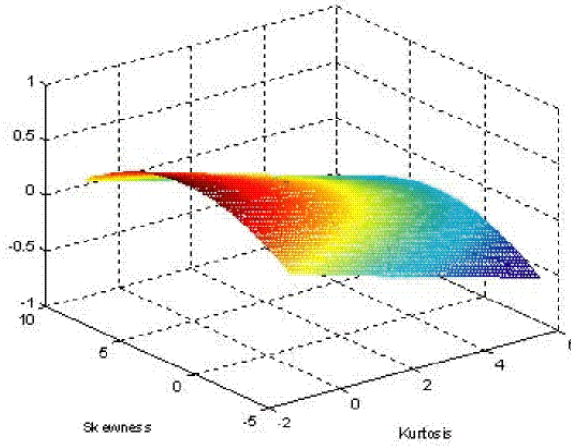


Figure 17: in-the-money put options ($X=107.5, S = 105$)

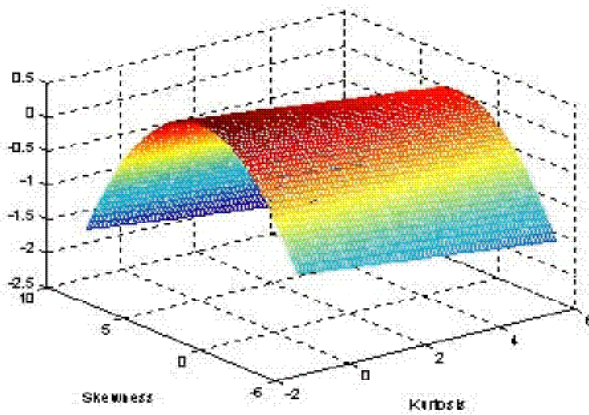
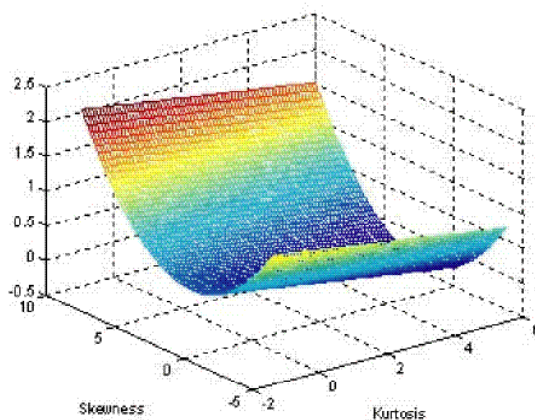


Figure 18: out-of-the money put options ($X=102.5$, $S = 105$)



V. Conclusions

The evidence presented in this paper suggests that considering the third and the fourth moments of the underlying return distribution produces significant price differences as compared to the Black-Scholes methodology. This pricing difference is largest in magnitude for out-of-the money call options and slightly in-the-money put options. Our results suggest option prices obtained using the Edgeworth expansion are lower than those computed using the Black-Scholes formula. When computing the “Greeks”, differences in magnitude are also substantial; Gamma under the two methods can differ by as much as 30 percent for both at-the-money calls and puts, with obvious implications for risk management. Sensitivity analysis revealed that varying both kurtosis and skewness can greatly affect the difference between Edgeworth and Black-Scholes option prices. As kurtosis increases and skewness decreases below zero, the pricing difference increases. This relationship is linear in kurtosis and non-linear in skewness. When skewness and kurtosis vary together, skewness dominates kurtosis for both in-the-money and out-of-the money puts and calls.

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