Incorporation of Learning Curves in Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) Models

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and

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Abstract:

This paper investigates the effect of learning curves on the basic Economic Order Quality (EOQ) and Economic Production Quality (EPQ) models of inventory management. It is well known that people show improvement when they do a task on a repetitive basis, and the time required to perform a task decreases due to learning. EOQ and EPQ models of inventory management existed for decades and so did formulas for learning curves. In this paper, we have incorporated learning curves in setup costs, and derived new formulas for the lot size and inventory costs. Further, comparisons of setup and inventory costs are illustrated with numerical examples. The learning effect reduces the average setup cost, and consequently the optimal lot size and total inventory cost become lower than those in the classical models. As a result of incorporating learning in the two models, the annual ordering or setup cost is ***not*** *equal to the annual inventory carrying cost, unlike the classical EOQ and EPQ models.*

Key Words: Inventory Management, Learning Curves, Economical Order Quantity, Economic Production Quantity

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Introduction:

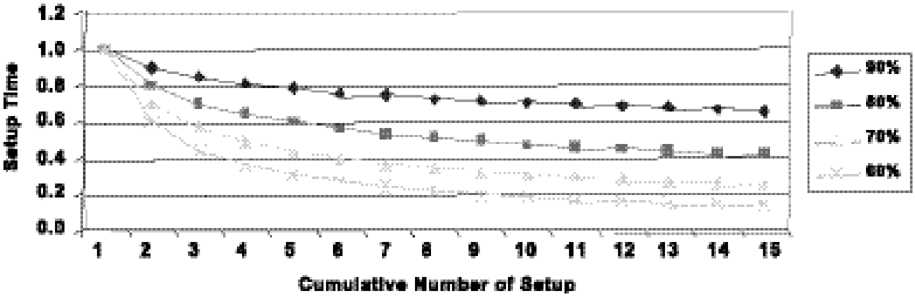
The Economic Order Quantity (EOQ) Model is a simple, inexpensive, and easy-to-use decision-making tool for minimizing inventory costs. However, the assumptions made in the formation of the basic EOQ model are rarely valid in practice. Osteryoung et al (1986) excellently enumerate and describe the usefulness, applicability, and benefits of EOQ models in the real-world and the article is worth perusing. The objective of this paper is to bring together researchers' results and practitioners' need and help them work together by extending the simple models in order to put them into practice and reap the benefit of research.

In this paper, we investigate the effect of reduced cost due to learning by virtue of repetitive setups on the basic Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models. Heizer and Render (2005, Page 362), Krajewski and Rotsman (2005, Page 659), Finch (2006, Page 399), Bowersox, Closs and Cooper (2002, Page289) describe different types of costs in inventory management models. Krajewski and Rotsman (2005) and Wisner, Leong, and Tan (2005) emphasize the importance of setup time reduction and illustrate its effect on inventory costs. Esrock (1985) discusses some strategies for reducing setup times, because lower setup costs lead to lower batch sizes in a production process. Sometimes, an investment is necessitated to buy new equipment, or to train workers on new methods that lead to reduction of setup costs. Porteus (1985) examines the trade-off between such investments and the benefits accruing from them.

Chand (1989) studied the effect of learning in setups and process quality. He combines the process quality with setups and that is different from out paper. Karwan, Mazzola, and Morey (1988) use learning curves in the production process also, whereas we study the effect of learning only on the setups. Spencer (1981)develops a model of competitive interaction and compares costs of individual firms.

With proper instruction and repetition, workers learn to perform their jobs more efficiently and effectively (Wisner, Leong and Tan, 2005), and consequently, direct labor hours per unit of a product are reduced. This learning effect can also be applied to perform setups in a production process. For example, Figure 1 depicts the learning effect that could have resulted from better work methods, tools, product design, or supervision, as well as from an individual's learning the task. For more details on learning curves, see Krajewski and Ritzman (2005) and Heizer and Render (2005, Page 574). Rana and Maheshwari (2001) describe how learning curves can be utilized in manufacturing processes, and break-even points can be lowered to make manufacturing companies and project managers more competitive.

Figure 1 : Reduction in Setup Time due to Learning



Replogle (1988) incorporates the learning effect due to repetitive setups and present a new EOQ model. His approach appears to be innovative and interesting. However, a careful perusal of that paper reveals that the function of the learning curve and the formulas derived as a result of the incorporation of the function are debatable for the accuracy. He used the formula for the nth setup as the average setup cost due to a learning effect, and that is incorrect. If Y_i is cost of the ith setup and a learning curve is utilized in setup costs, then obviously the following is true.

$$Y_1 > Y_2 > Y_3 > \ldots\ldots > Y_{n-1} > Y_n > Y_{n+1}.$$

The cost of the nth setup is less than (n-1)th setup as well as the cost of all previous setups. Therefore, the average setup cost is always higher than the cost of the nth setup. Raplogle's assumption is that he used the cost of the nth setup equal to the average setup cost in his analysis of the EOQ model and in deriving other formulas of lot size and total cost. As a matter of fact, by assuming the average setup cost equal to the cost of the nth setup, the analysis of annual setup cost and the total annual costs including inventory carrying cost becomes simple. However, in reality, the annual setup cost is the sum of all annual setups. Note that an individual setup cost is a decreasing function due to a learning effect. Also, we include in the paper the incorporation learning curves in Economic Production Quantity (EPQ) models as well, which the other authors had not done.

In this paper, we present: (a) the incorporation of learning effect in the EOQ and the EPQ models, (b) its effect on the relevant inventory costs, and (c) the results of our analysis. For the ease of comparison, the notations and values of the parameters used in this paper are the same as used by Replogle (1988). The idea of learning curves is not new, neither the EOQ and EPQ models. However, the formulas presented in this paper by employing learning curves in the basic EOQ and EPQ models and the analyses of the example problems are entirely new and are our contributions.

It is well known that because of the learning effect, the time required to perform a task is reduced when the task is repeated. Applying this principle, it may be assumed that the time required to perform the n+1st setup will always be less than that for the nth setup and that the time required will decrease at a declining rate as cumulative number of setups increase. This reduction in time for setups follows an exponential function as show in Figure 1. Industrial engineers have observed that the learning rate ranges from 70% to 95% in the manufacturing industry. For example, in a batch production scenario an 80% learning curve denotes a 20% reduction in the setup time with each doubling of repetitions. A 100% curve would imply no improvement at all.

Notations

D = annual demand for a product,
S= cost to place the first order or to perform the first setup,
C = inventory carrying per unit per year,
Q = lot size,
TIC = Total inventory related cost,
Y_n = cost to perform the nth setup,
n = cumulative number of orders or setups,
r = the learning rate, a lower rate implies faster learning,
(for 90% and 80%, r = 0.9 and r = 0.8, respectively)
b = Ln(r)/Ln (2).

For the classical EOQ model, Q^* is the optimal batch size that minimizes TIC, and TIC^* is the minimal value of TIC. The classical formulas are:

$$Q^* = \text{Sqrt}\ (2DS/C) \quad (1)$$

$$TIC = \frac{QC}{2} + \frac{DS}{Q} \quad (2)$$

$$TIC^* = \text{Sqrt}\ (2DSC) \quad (3)$$

$$Y_n = S\, n^b \quad (4)$$

Incorporation of Learning Curve in The EOQ Model

Let S_n be the cost of the first n setups, then

$$S_n = \sum_{k=1}^{n} S\, k^b, \text{ Or } S_n = S \sum_{k=1}^{n} k^b \quad (5)$$

For example, an 80% learning curve yields b = Ln(0.8)/Ln(2) = -0.3219. For other learning curves, values of b are listed in Table 1. The number of annual setups will be:

$$n = D/Q \quad (6)$$

As seen from above in equation (5), b is a fractional number between -1 and 0. The right hand side equation (5) is series, and mathematically, its solution does not exit in a close form. In order to obtain its solution, we assume that for the kth setup,

$$k^b = \int_{k-0.5}^{k+0.5} k^b \, dk \qquad (7)$$

Also, we know that $1^b = 1$ and $2^b = r$, the learning rate. Now, after substituting equation (7) in equation (5), the following expression is obtained:

$$S_n = S\left[1 + r + \int_{3-0.5}^{n+0.5} k^b \, dk\right] \qquad (8)$$

$$\text{Or } S_n = S[1 + r + (n + 0.5)^{b+1} - 2.5)^{b+1})/b+1] \qquad (9)$$

Substituting equations (6) and (9) in equation (2), we get:

$$TIC = QC/2 + S[1 + r + (D/Q + 0.5)^{b+1} - 2.5)^{b+1})/(b+1)] \qquad (10)$$

In order to determine an optimal value of Q (i.e., Q^*), the first derivative of TIC with respect to Q should be equated to zero. And the expression obtained, after an algebraic manipulation, leads to the following equation:

$$(2SD)/C = Q^2 (D/Q+0.5)^{-b} \qquad (11)$$

The value Q that equates the right and left hand sides of equation (11) would be the optimal batch size. Unfortunately, equation (11) does not yield a close form solution. Nevertheless, equation (11) can easily be solved by using a trail-and-error method or the GOAL SEEK function of the Microsoft Excel. When n is sufficiently large (say, 10 or more), another approximation of S_n is:

$$S_n = S (1 + r + 3^b (1+b/2) + (n^{b+1} - 3^{b+1})/(b+1) \qquad (12)$$

After substituting equation (11) in equation (2), we get:

$$TIC = Q\,C/2 + S\,[1 + r + 3^b\,(1+b/2) + \{(D/Q)^{b+1} - 3^{b+1}\}/(b+1)] \qquad (13)$$

The calculus method of minimization of TIC from equation (13) yields.

$$Q^* = (2\,S\,D^{\,b+1}/C)^{1/(b+2)} \qquad (14)$$

For no learning at all, the learning rate is 100%, i.e. r = 1 and b = 0.

Therefore, for the learning rate of 100%, equations (10) and (13) would reduce to equation (2) and equations (11) and (14) would become the same as equation (1).

Having derived equation (11) and (14), we can now show numerically with an example the effects of learning curves on the lot size and inventory costs.

An Example and Numerical Comparison

In our example, the parameter values are assumed to be the same as in the Replogle (1988). The values of the parameters are as follows:

Annual demand, D = 10,000 units,

Carrying cost, C = $1 per unit per year,

Cost of the first order, S=$12,

Learning rate=80%, i.e. r = 0.8 and b = -0.3219

The **GOAL SEEK** function of an electronic spreadsheet such as Microsoft Excel is utilized to compute the value of Q^*, the optimal lot size. Equation (11) is used for the computation of Q^*. In one cell, the value of the left hand side, (2SD)/C, is computed, and in another cell, the formula for the right side is entered by referring to a third cell which contains a numerical value of Q, say 100. The optimal value of Q is then obtained by using the **GOAL SEEK** function of Microsoft Excel. The function itself obtains the desired value of Q such that the two cells containing the computed values of the left and right hand sides become equal. This value of Q is the Q*. In this example, values of Q^* obtained from equations (11) and (14) are 273.954 and 274.669 units, respectively. Even though the latter value is obtained from a formula that provides an approximate value of S_n, it is quite close (less than 0.05%) to the value of Q^*. For practical purposes, either of the equations, (11) or (14) is recommended. Since determining Q^* from equation (11) is also quite easy, we will use Q^* = 273.954 = 274 in our analysis.

For comparison, optimal lot sizes, inventory costs, and percentage reduction due to learning rates of 90% to 60% are placed in Table 1, which also includes these values with no learning. It is essential to note that due to the incorporation of the learning curve in the EOQ model, the annual inventory carrying cost and the ordering cost for an optimal lot size are not equal. Whereas in the basic EOQ model, the inventory carrying cost must be equal to the annual ordering cost for an optimal lot size. Table 1 clearly demonstrates these facts. For a 90% learning rate, the annual inventory carrying cost is $190.83, the annual ordering cost is $221.36, and the total cost is $412.19 at an optimal lot size of 381.67. However in a classical EOQ model, these costs are $219.09, $219.09, and $438.18 respectively, at an optimal lot size of 547.72. The same costs and optimal lot sizes are also listed for 80%, 70%, and 60% learning rates.

A closer look at Table 1 reveals that the higher the learning rate (60% learning is higher than 70%) the lower is the optimal lot size, and hence lower the annual inventory carrying cost. It can also be noted that the annual ordering cost is also steadily decreasing with higher learning rates. The percentage reduction in costs is also provided in Table 1. For 80% learning, we computed the lot size at which the annual ordering and inventory holding costs become equal. The lot size is 335.22 units. Consequently, the total cost rises to $335.22, which is higher than the minimal cost of $330.41.

Table 1: A Comparison of Optimal Lot Sizes and Inventory Costs for the EOQ model
Assumed parameters are: Annual Demand = 10,000
Carrying Cost = $1 per unit per year, Ordering Cost = $12/order

EOQ Model with Different Learning Rates				
Learning Rate	90 %	80 %	70 %	60 %
b = Ln r/Ln 2	-0.1520	-0.3219	-0.5146	-0.7370
Q* Based on Equation (11)	381.67	273.95	171.81	84.09
Q* Based on Equation (14)	382.26	274.67	172.32	84.29
Annual Carrying Cost	$190.83	$136.98	$85.91	$42.04
Annual Ordering Cost	$221.36	$193.44	$160.32	$121.66
Total Annual Cost	$412.19	$330.41	$246.23	$163.70
Percent Reduction of Carrying Cost	22.09	44.08	64.93	82.84
Percent Reduction of Ordering Cost	9.63	21.03	34.55	50.33
Percent Reduction of Total Cost	15.86	32.55	49.74	66.58
Classical EOQ Model Without Learning Effect				
Q*	489.90			
Annual Carrying Cost	$244.95			
Annual Ordering Cost	$244.95			
Total Annual Cost	$489.90			

Assuming the cost of the first order is $1 and using the learning curve of 80%, costs of placing the first 4 orders are:

$Y_1 = \$_1$, $Y_2 = \$0.80$, $Y_3 = \$0.702$, $Y_4 = \$0.64$

Using these four costs, the average cost of the first four orders equals to $0.7855. In equation (4), notice that Y_n is the cost of the nth setup and not the average cost of n setups (n denotes the number of all setups). Replogle (1998) used the average cost equal to $0.64, and not $0.7855, the correct average cost. Using Yn as an average cost implies much faster learning than the stated value (80% used in his calculations). For example, an average cost of 0.64 for the first four units can only be achieved if the learning rate, r, is 0.645. As can be seen from Table 1, the lot size is reduced to 274 units, a reduction of 44%, as against 55% stated by Replogle (1988). In the classical EOQ model, the average ordering cost is the same as the cost of the first order, whereas due to the learning effect, the average ordering cost is always less than the cost of the first order. Therefore, the faster the learning (i.e., lower value of r), the lower the average ordering cost, and hence the lower the lot size. The 55% reduction in the lot size was merely due to the implied faster learning than the assumed 80% as stated by Replogle (1988). Table 1 provides detailed information on optimal lot size, ordering, inventory carrying, and total costs, as well as on the percentage reduction of these costs.

The above example is an illustration of the "fixed lot size." Another approach could be to use variable lot sizes. Since the cost of a subsequent order is reduced, so does the lot size. The question now arises how to determine these different lot sizes so that the annual inventory cost is minimized. The problem involving different lot sizes is quite complex and is beyond the scope of this paper. Nevertheless, we attempted to determine lot sizes by using the following formula:

$$Q^*_n = \text{Sqrt}\,(2DS_n/C)$$

Where, Q_n^* is nth lot size and S_n the cost of nth order. The first lot size would be 490 units, the second 438, the third 410 and so forth for an 80% learning. The lot sizes so computed would make inventory carrying and ordering costs equal for each lot size, and the total annual cost comes out to be $337.42. As already stated, the lot size that makes the two costs equal is not optimal.

The EPQ Model

In the basic EOQ model, one assumption is that replenishment of quantity Q is instantaneous. However, there are instances when a firm is both the producer and consumer of a product, or deliveries are spread over time, i.e., replenishments are non-instantaneous and inventories buildup gradually. The model for this type of situation is known as the Economic Production Quantity (EPQ) model. Normally, production occurs only during a part of the inventory cycle, whereas consumption or usage takes place over the entire cycle. Therefore, inventory accumulation rate is equal to the difference between the production and the consumption rates of the product.

For the EPQ model, if P is the production rate per year, then formulas for Q^* and TIC are as follows:

$$Q^* = \text{Sqrt}\ (2DS/(1-(D/P)C), \quad (15)$$

$$TIC = Q(1-D/P)C/2 + (D/Q)S, \quad (16)$$

$$TIC^* = \text{Sqrt}\ (2DS\ (1-(D/P)C). \quad (17)$$

Incorporation of Learning Curve in The EPQ Model

The sum of the setup costs given by equation (8) remains the same. With the incorporation of learning curve, we get the following formulas:

$$TIC = Q\ C(1-D/P)/2 + S\ [1+ r + \{(D/Q+0.5)^{b+1} -(2.5)^{b+1}\}/b+1\}] \quad (18)$$

And

$$2SD/(1-D/P)C = Q^2(D/Q+0.5)^{-b} \quad (19)$$

Equations (18) and (19) are equivalent and similar to equations (10) and (11): the only difference is that C is replaced by (1-D/P)C. Notice that the same difference is in the classical EOQ and EPQ equations also. If the approximation of S_n, as given by equation (12) is used, then the following equations are obtained:

$$TIC = QC(1-D/P)/2 + S\,[1 + r + 3^{b}\,(1+b/2) + \{(D/Q)^{b+1} - (3)^{b+1}\}/b+1] \qquad (20)$$

$$Q^{*} = [\{2\,S/((C)(1-D/P))\}\ \ D^{b+1}]^{1/(b+2)} \qquad (21)$$

Again, equations (20) and (21) are equivalent to equations (13) and (14) of the EOQ model. As it was observed in the EOQ model, here also, with no learning effect, inventory cost equations (18) and (20) are reduced to equation (16), and the optimal lot size equations (19) and (21) are reduced to equation (15).

In order to illustrate numerically the effect of learning on the EPQ model, the production rate per year is assumed to be 50, 000 units and the values of D, C, S, and r are kept the same as used in the example for EOQ model. The optimal lot sizes computed by using equations (19) and (21) are 312.80 and 313.73 for an 80% learning rate, respectively. For practical purposes, either of equations (19) and (21) can be used, because the difference is less than 0.05%. However, equation (19) is more accurate. Table 2 shows the comparison of costs and lot sizes with and without the learning effect.

Table 2: A Comparison of Optimal Lot Sizes and Inventory Costs for the EPQ model

Assumed parameters are:

Annual Demand = 10,000 units, Annual Production Rate = 50,000 units
Carrying Cost = $1 per unit per year, Ordering Cost = $12/order

EOQ Model with Different Learning Rates				
Learning Rate	90 %	80 %	70 %	60 %
b = Ln r/Ln 2	-0.1520	-0.3219	-0.5146	-0.7370
Q* Based on Equation (11)	430.57	312.80	199.57	100.29
Q* Based on Equation (14)	431.3	313.7	200.3	100.6
Annual Carrying Cost	172.23	125.12	79.83	40.12
Annual Ordering Cost	159.60	140.85	118.33	91.54
Total Annual Cost	331.82	265.98	198.16	131.65
Percent Reduction of Carrying Cost	21.39	42.89	63.56	81.69
Percent Reduction of Ordering Cost	27.16	35.71	45.99	58.22
Percent Reduction of Total Cost	24.27	39.30	54.78	69.95
Classical EOQ Model Without Learning Effect				
Q*	547.72			
Annual Carrying Cost	$219.09			
Annual Ordering Cost	$219.09			
Total Annual Cost	$438.18			

For a classical EPQ model, the optimal lot size is 547.72 and annual setup, inventory holding, and total costs are $219.09, $219.09, and $438.18 respectively. For a 90% learning rate, the optimal lot size is reduced to 431.3, and the three costs are \$159.60, \$172.23, and \$331.82, respectively. Table 2 reveals that the total annual costs are reduced by 24.27%, 39.30%, 54.78%, and 69.95% for 90, 80, 70, and 60 percent learning rates respectively. In EPQ model also, we can conclude that the higher the learning rate, the higher the savings in annual costs.

Sensitivity Analysis

A sensitivity analysis can yield insights into the management of inventory and the effect on batch size and inventory cost, when parameters change.

Effect on Lot Size

Although equation (14) yields Q^* based on an approximation of S_n, for practical purposes it is accurate enough for employing in the sensitively analysis. From equation (14), it is obvious that:

$$Q^* \text{ proportional to } D^{(b+1)/(b+2)} \tag{22}$$

For a 100% learning curve (i.e., no learning effect), the optimal lot size increases in proportion to the square root of the annual demand. For a learning curve of 80% and 70%, the exponents of D are reduced to 0.4 and 0.327, respectively. Based on our computations, it is concluded that the higher the learning rate the lesser sensitive Q^* is to the demand rate. It is also observed that:

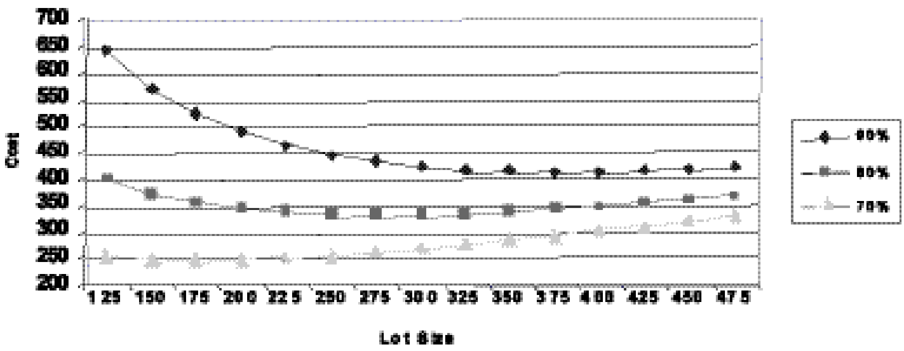
$$Q^* \text{ proportional to } S^{1/(b+2)} \tag{23}$$

The exponents of S are 0.59 and 0.67 for 80% and 70% learning curves respectively, as against 0.5 for no learning. It means that the optimal size becomes more sensitive with a faster learning rate. The effect on Q^* due to a variation in the carrying cost is similar to that due to the setup or ordering cost. The only difference is that Q^* is inversely proportional to $C^{1/(b+2)}$, whereas Q^* is directly proportional to $S^{1/(b+2)}$.

Effect on Cost

The equation (10) on inventory cost is a function of Q, C, S, and b. Figure 2 illustrates how TIC would vary with different lot sizes. It is important to note that the graph of TIC versus Q has a similar shape as the classical EOQ model. However, a negative deviation of lot size from Q^* is more disadvantageous and expensive than the positive deviation. Fro example, a smaller lot size of 200 units, optimal being 274 units, would cost $11.10 higher than the optimal cost of $330.43. Whereas the lot size of 348 units (the same deviation on the positive side) would cost $7.12 more than the optimal cost. Although a smaller lot size results in more setups and hence more repetitions and less inventory carrying cost, the savings are not sufficient enough to compensate for the additional cost due to more setups. Nevertheless, the offset due to the deviation of lot size from the optimal value is dependent on the value, say, $5 per unit per year, the optimal lot size will be smaller than 274 units, and the inventory cost will increase.

Figure2: Lot Size Versus Annual Cost with Learning Effects



Conclusion

This paper illustrates the effect of learning on the basic EOQ and EPQ models. Our procedures extend the basic models that can be used for realistic inventory problems, because the setup cost is reduced in practice as the cumulative number of setups increases. The incorporation of the learning effect reduces both the lot size and inventory costs. Due to learning effect, the inventory carrying and setup costs are not equal for an optimal lot size unlike the classical models. For no learning, our formulas are reduced to the basic EOQ and EPQ formulas. Our calculations in Tables 1 and 2 are based on the formulas derived in this paper. We created a Microsoft Excel worksheet, attached as Table 3, and the calculations can be extended to any learning rates.

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